NAG Toolbox for MATLAB

f07gr

1 Purpose

f07gr computes the Cholesky factorization of a complex Hermitian positive-definite matrix, using packed storage.

2 Syntax

```
[ap, info] = f07gr(uplo, n, ap)
```

3 Description

f07gr forms the Cholesky factorization of a complex Hermitian positive-definite matrix A either as $A = U^{\mathrm{H}}U$ if **uplo** = 'U' or $A = LL^{\mathrm{H}}$ if **uplo** = 'L', where U is an upper triangular matrix and L is lower triangular, using packed storage.

4 References

Demmel J W 1989 On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'

The upper triangular part of A is stored and A is factorized as $U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'

The lower triangular part of A is stored and A is factorized as LL^{H}, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n – int32 scalar**

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **ap**$(*)$ **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The n by n Hermitian matrix A, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i+j(j-1)/2)$ for $i \leq j$;
if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i+(2n-j)(j-1)/2)$ for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ap**$(*)$ **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

If **info** = 0, the factor U or L from the Cholesky factorization $A = U^{\mathrm{H}}U$ or $A = LL^{\mathrm{T}}$, in the same storage format as A.

2: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **info**.

info > 0

If **info** $= i$, the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A. To factorize a Hermitian matrix which is not positive-definite, call f07pr instead.

7 Accuracy

If **uplo** = 'U', the computed factor U is the exact factor of a perturbed matrix $A+E$, where

$$|E| \leq c(n)\epsilon \left|U^{\mathrm{H}}\right||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If **uplo** = 'L', a similar statement holds for the computed factor L. It follows that $\left|e_{ij}\right| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

A call to f07gr may be followed by calls to the functions:

f07gs to solve $AX = B$;

f07gu to estimate the condition number of A;

f07gw to compute the inverse of A.

The real analogue of this function is f07gd.

9 Example

```
uplo = 'L';
n = int32(4);
ap = [complex(3.23, +0);
     complex(1.51, +1.92);
     complex(1.9, -0.84);
     complex(0.42, -2.5);
     complex(3.58, +0);
     complex(-0.23, -1.11);
     complex(-1.18, -1.37);
     complex(4.09, +0);
     complex(2.33, +0.14);
     complex(4.29, +0)];
[apOut, info] = f07gr(uplo, n, ap)
```

```
apOut =
   1.7972
   0.8402 + 1.0683i
   1.0572 - 0.4674i
   0.2337 - 1.3910i
   1.3164
  -0.4702 + 0.3131i
   0.0834 + 0.0368i
   1.5604
   0.9360 + 0.9900i
   0.6603
info =
           0
```